B.A./B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH6DSE32

(Industrial Mathematics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Write brief note on seismic image.

(b) Why do we require attenuated Radon transform?

(c) State one result of Fourier Slice theorem.

(d) Find the inverse Fourier transform of a step wave.

(e) Define band limited function. Where is it used?

(f) Plot the Fourier transformation of $f(x) = \sin(ax), |a| > 0$.

(g) What is the use of a space of bounded measurable functions?

(h) What are the applications of Sheep-Logan filter?

(i) Find the Radon transformation of the body given by

$$E(x,y) = \begin{cases} 1, & 1 \le x^2 + y^2 \le 9 \\ -1, & everywhere\ else. \end{cases}$$

(j) Why do we need to include the idea of Affine space? Explain with examples.

(k) What do you mean by low pass cosine filter?

(l) Where do you need point spread function?

(m) Which kind of tomography is important to study back projections?

(n) Find the area under the curve of Dirac delta function.

(o) State an application X-ray tomography.

2. Answer *any four* questions: 5×4=20

(a) Define the characteristic function of the interval $[-l, l]$. Find the Fourier sine transformation of this function. 2+3

(b) What is convolution back projection? Write down the optoacoustic tomography for this back projection. 2+3

(c) State and prove Nyquist's theorem.

(d) Applying the scaling property of the ideal ramp filter, find out fan-beam reconstruction formula.

(e) Evaluate the Fourier transform of weighted projections for each parametric change.

(f) Apply the Rayleigh-Plancherel theorem to the function $f(x) = e^{-|x|}$ in order to evaluate the integral

$$\int_{-\infty}^{\infty} \frac{1}{(1+\omega^2)^2} d\omega \quad .$$

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3. Answer *any two* questions: 10×2=20

(a) (i) Give an example of two random variables that are individually Gaussian distributed but their joint distribution is not Gaussian. Give proper justification.

(ii) If $A(\omega) = |\omega|.\left(\sin\frac{\left(\frac{\pi\omega}{2L}\right)}{\frac{\pi\omega}{2L}}\right).\Pi_L(\omega)$, then prove that

$$(\mathcal{F}^{-1}A)\left(\frac{n\pi}{L}\right) = \frac{4L^2}{\pi^3(1-4n^2)}.$$

5+5

(b) (i) Show that the rotation vector ω can be defined as

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \partial_y u_z & - & \partial_z u_y \\ \partial_z u_x & - & \partial_x u_z \\ \partial_x u_y & - & \partial_y u_x \end{pmatrix}.$$

Hence define tilt. What are its physical significances?

(ii) Write and explain the algorithm of CT scan. How can data be derived from a century old mummy using CT scan, keeping it intact? Answer briefly with help of mathematical steps. 5+(2+3)

(c) (i) Show that the functions $F_1(t) = e^{(\alpha+i\omega)t}$ and $F_2(t) = e^{(\alpha-i\omega)t}$, where α and ω are real constants, satisfy the (second-order linear) differential equation

$$y'' - 2\alpha y' + (\alpha^2 + \omega^2)y = 0.$$

Using Euler's formula, show that the functions $y_1 = e^{\alpha t}\cos(\omega t)$ and $y_2 = e^{\alpha t}\sin(\omega t)$, also satisfy the same differential equation.

(ii) Define f by $f(x,y) = \begin{cases} x, x^2 + y^2 \leq 1 \\ 0, x^2 + y^2 > 1 \end{cases}$. Compute the Radon transformation of $f\left(\frac{1}{2}, \frac{\pi}{6}\right)$.

5+5

(d) (i) Suppose that a Hanning window is applied to the ramp filter, using Fourier transform properties, find analytically the impulse responses.

(ii) What is Hounsfield unit of a tissue? Explain with plots.

5+5